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مراجعة

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Theoretical Questions
Statics

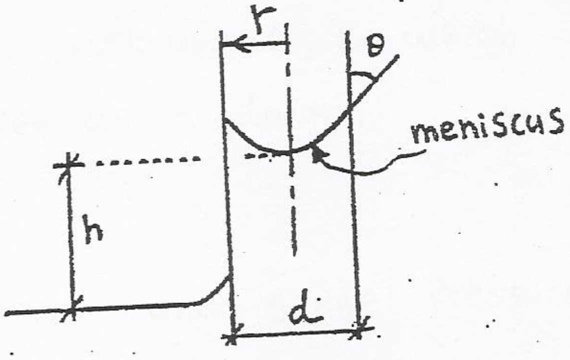
1. Derive the equation required to calculate the capillary rise in a capillary tube Prove
2. Derive the equation required to calculate the pressure inside a water drop Prove
3. Define Newtonian and Non-Newtonian fluids
4. Define gauge and absolute pressure, use a sketch to illustrate their relation.
5. Write short notes on:
   a) Mercury barometer b) Aneroid Barometer c) Bourdon gage
6. What is the difference between:-
   a) Piezometer b) Manometer c) Differential manometer
7. What is the pressure head? How pressure can be expressed in terms of height of a liquid column?
8. What are the disadvantages of using a piezometer
9. When do we use an inverted differential manometer
10. Prove that the intensity of pressure at any point in a fluid at rest, is the same in all directions Prove
11. What is meant by the intensity of pressure? How does it vary with the depth of fluid. Prove
12. For a submerged inclined plane surface, determine the resultant hydrostatic force due to the liquid acting on one side of the surface and its line of action Prove هام
13. Explain clearly how do you calculate the resultant force acting on a concave cylindrical surface submerged in a liquid
14. Apply the basic hydrostatics equation to determine the pressure variation in the horizontal and vertical directions and the slope of the surface of constant pressure for any body fluid in rigid body motion. Prove
15. State Archimedes Principle
16. Define the following:-
    a) Center of Buoyancy b) Metecenter c) Metacentric Height
17. State the types of Equilibrium of floating bodies.
    Define each one using a neat sketch

1- Derive the equation required to calculate the capillary rise in a capillary tube

The liquid column is forced up until its weight is balanced by the Force

$$\sigma(\pi d)\cos\theta = \frac{\pi d^2}{4} * h * \gamma$$

$$\boxed{h = \frac{4\sigma\cos\theta}{\gamma d}}$$



2- Derive the equation required to calculate the pressure inside a water drop

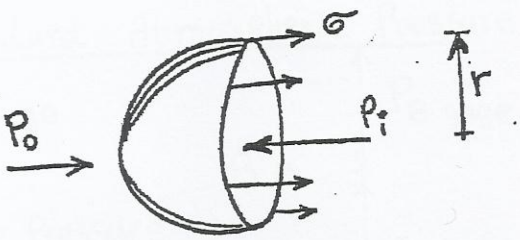
Water droplet

$$(P_i - P_o)\pi r^2 = \sigma(2\pi r)$$

$$P_i - P_o = \frac{2\sigma}{r}$$

or

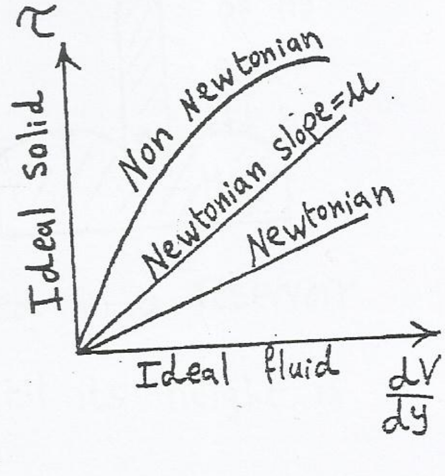
$$\boxed{\Delta P = \frac{4\sigma}{d}}$$



3- Define Newtonian and Non-Newtonian fluids

Newtonian Fluids e.g. oil, air, water

* They are fluids with Constant Viscosity
* Shear Stress is linearly dependant on velocity gradient



Non- Newtonian Fluids e.g. Ink, Paint, blood

- Viscosity of these fluids changes with velocity
- Shear Stress is not linearly dependant on the velocity gradient

4- Define gauge and absolute pressure, use a sketch to illustrate their relation.

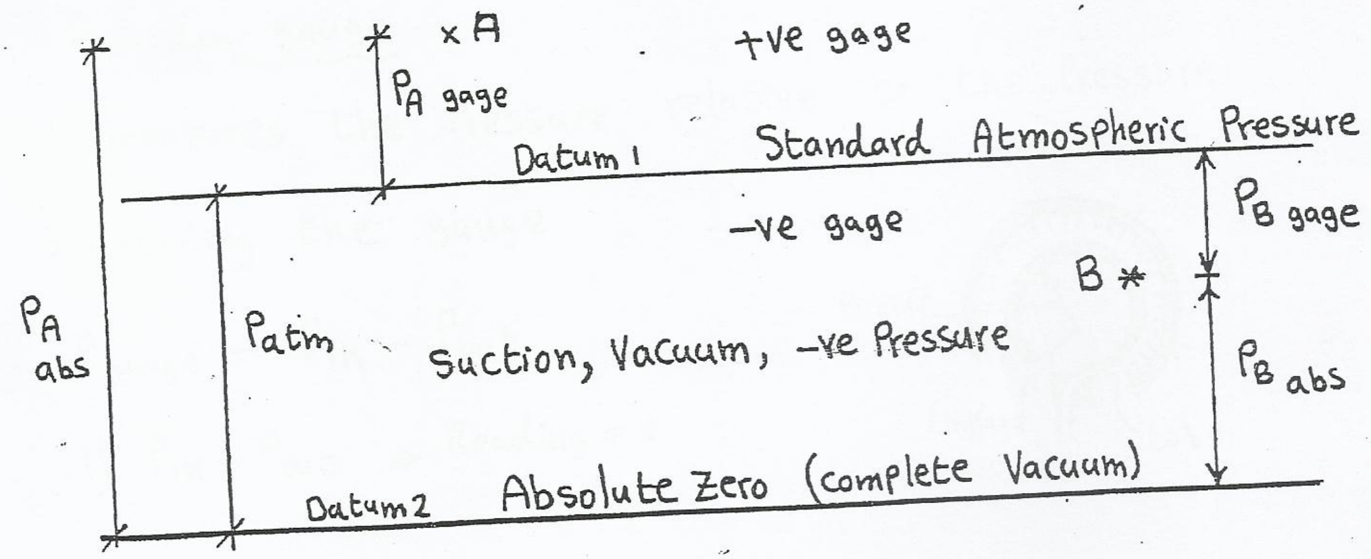
Gauge Pressure

It is the pressure measured by an instrument, in which the atmospheric pressure is taken as a datum

Absolute Pressure

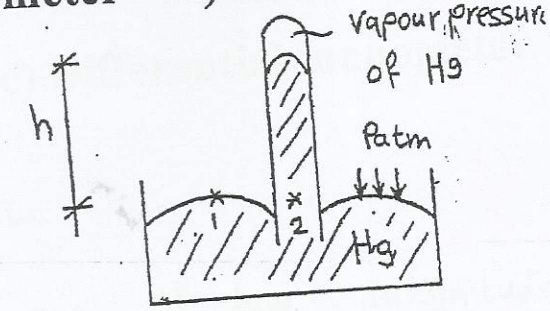
It is the sum of the atmospheric and gauge Pressure

$$\boxed{P_{abs} = P_{atm} + P_{gage}}$$



5- Write short notes on:
a) Mercury barometer b) Aneroid Barometer c) Bourdon gage

Mercury Barometer

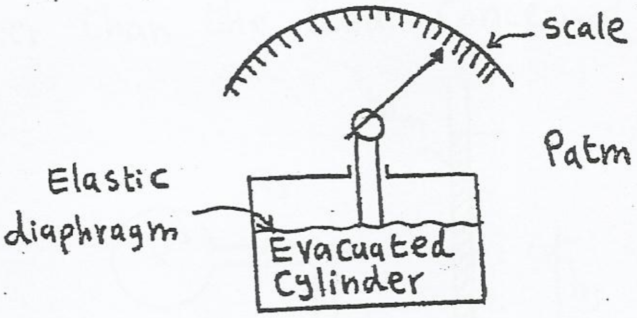


* It measures the atmospheric Pressure in absolute units
* When a tube filled with Mercury is inverted in a reservoir filled with Mercury, the Mercury drops until its height is balanced by the atmospheric Pressure

$$P_{atm} = \gamma_{Hg}\, h$$

Aneroid Barometer

It measures the diffrence between the atmospheric Pressure and an evacuated Cylinder

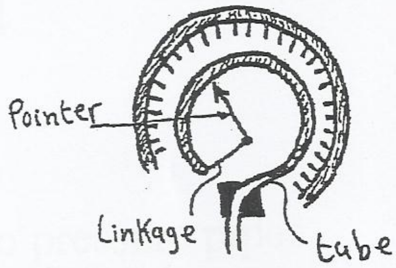


Bourdon gauge

It measures the Pressure relative to the Pressure Surrounding the gauge

$P_{gauge} = P_{in} - P_{out}$

If $P_{in} = P_{out} \Rightarrow$ Reading $= 0$



6- What is the difference between:-

a) Piezometer **b) Manometer** **c) Differential manometer**

Piezometer

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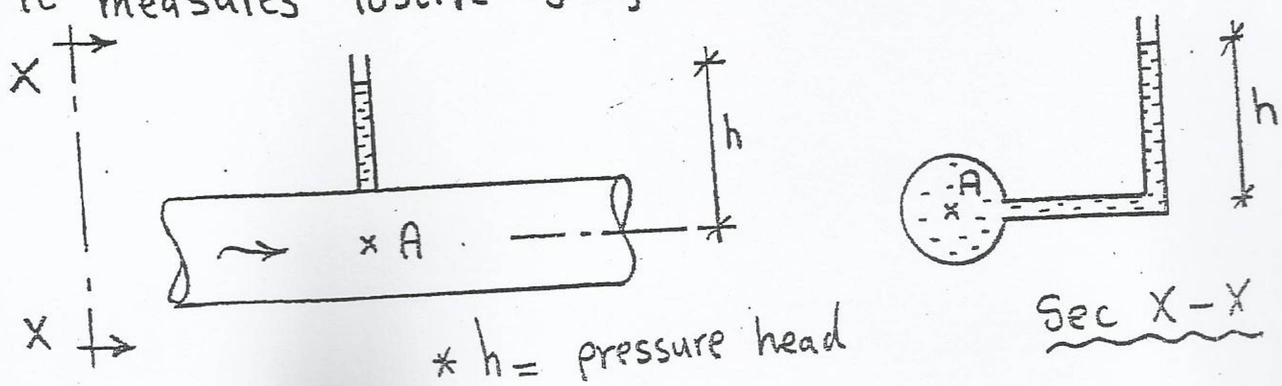
It measures Postive gauge Pressures of low magnitudes

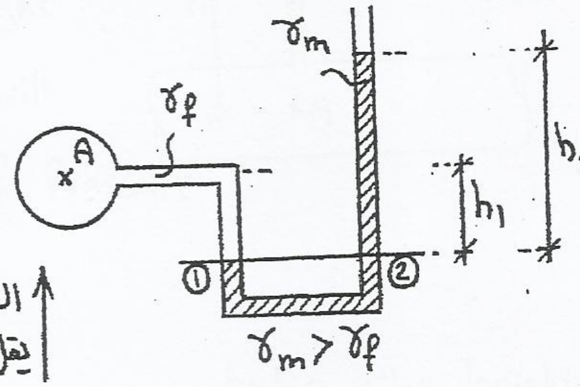


* h = pressure head

Manometers استخدام سائلين

It measures fluid pressures by using different fluids which may be heavier or lighter than the fluid concerned

Simple manometer



$P_1 = P_2$

$P_1 = P_A + \gamma_f h_1$

$P_2 = P_{atm} + \gamma_m h_2$

$\Rightarrow P_A = P_{atm} + \gamma_m h_2 - \gamma_f h_1$

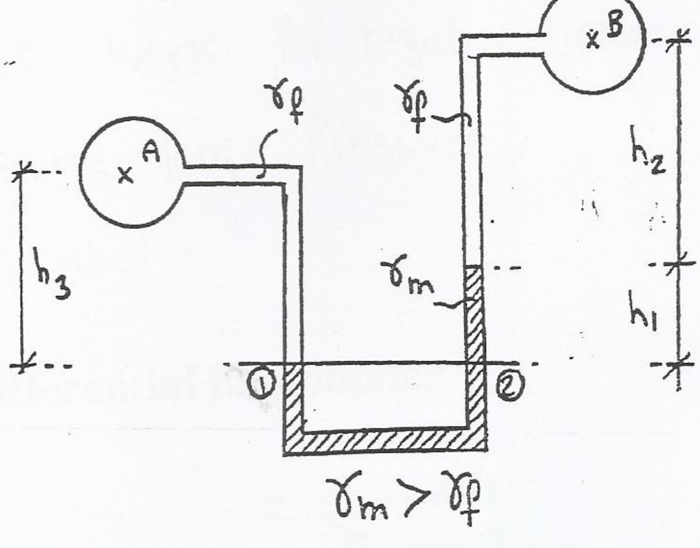
الضغط يقل (−)

الضغط يزداد (+)

Differential manometers

It measures the difference between two pressure pipes

Differential manometer



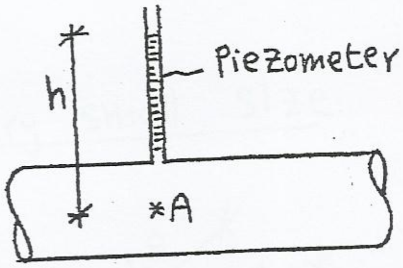
$P_1 = P_2$

$P_1 = P_A + \gamma_f h_3$

$P_2 = P_B + \gamma_f h_2 + \gamma_m h_1$

7- What is the pressure head? How pressure can be expressed in terms of height of a liquid column?

Pressure head



Pressure head is the height of a column of fluid that will produce the given intensity of Pressure

$h = \frac{P}{\gamma}$ = Pressure head

When a Piezometer is inserted in a tube the height of which the fluid rises is the Pressure head.

8- What are the disadvantages of using a piezometer

Limitations

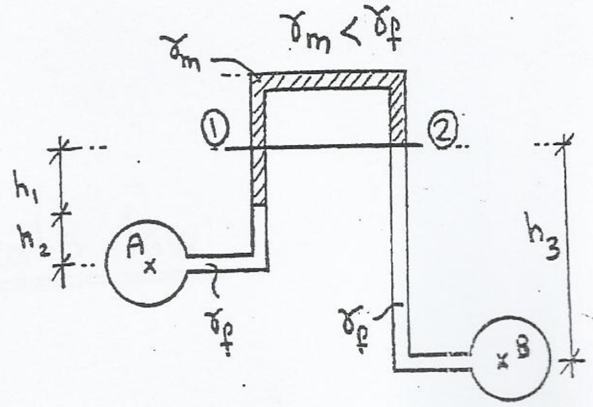
a- Piezometers does not work for negative Pressures

b- It is impractical to mesaure large Pressure (we need a very long tube)

9- When do we use an inverted differential manometer

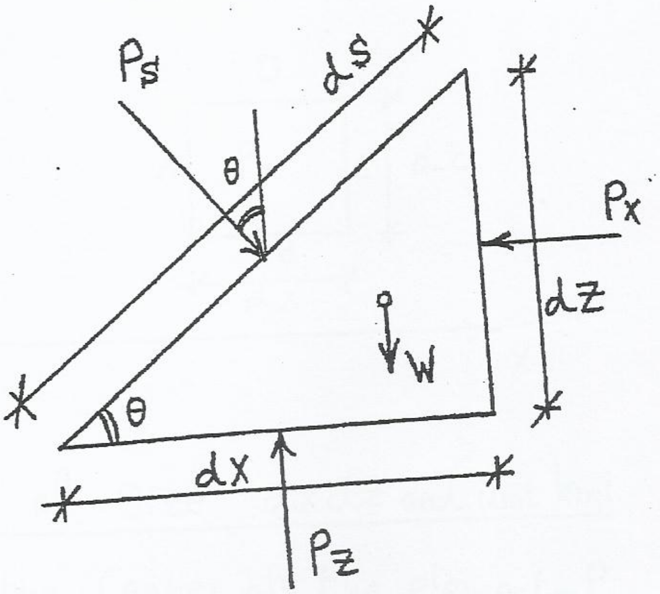
Inverted U-tube manometer is used

- When there is a small pressure difference
- A light liquid such as oil is used.



10- Prove that the intensity of pressure at any point in a fluid at rest, is the same in all directions

Consider a triangular prism of very small size



$\Sigma F_x = 0$

$P_s . ds \sin\theta = P_x . dz$

$P_s . \cancel{ds} \frac{\cancel{dz}}{\cancel{ds}} = P_x . \cancel{dz}$

$\therefore \boxed{P_s = P_x}$

$\sin\theta = \frac{dz}{ds}$

$\cos\theta = \frac{dx}{ds}$

$\Sigma F_z = 0$

$P_s . ds \cos\theta + W = P_z . dx$

$P_s . \cancel{ds} . \frac{\cancel{dx}}{\cancel{ds}} + \frac{1}{2} \cancel{dx} dz \gamma = P_z . \cancel{dx}$

$P_s + \frac{1}{2} dz \gamma = P_z$

$\hookrightarrow dz \simeq 0$

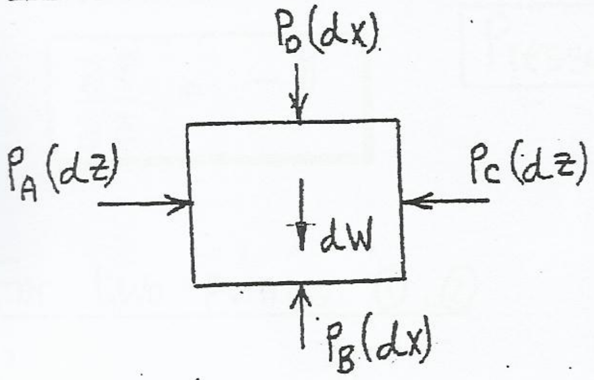
$\boxed{P_s = P_z}$

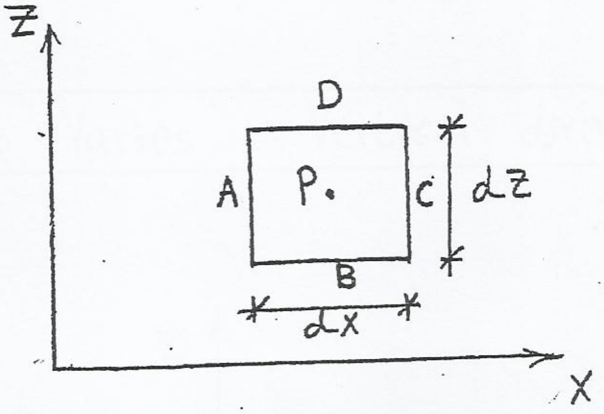
$\boxed{P_x = P_z = P_s}$ Pascal's law

11- What is meant by the intensity of pressure? How does it vary with the depth of fluid.

Intensity of Pressure means rate of change of Pressure in a certain direction $\left(\frac{\partial P}{\partial x}, \frac{\partial P}{\partial z}\right)$

<u>Variation of Pressure</u>





<u>Consider a fluid element of size $dx\,dz$ and unit lengt</u>

Let the Static pressure at the center of the element $= P$

$$\Sigma F_x = P_A(dz) - P_c(dz) = 0 \quad \text{......(1)}$$

$$\Sigma F_z = P_B(dx) - P_D(dx) - dW = 0 \quad \text{......(2)}$$

$$\because P_A = P - \frac{\partial P}{\partial x}\cdot\frac{dx}{2}, \qquad P_c = P + \frac{\partial P}{\partial x}\cdot\frac{dx}{2}$$

$$P_B = P - \frac{\partial P}{\partial z}\cdot\frac{dz}{2} \qquad P_D = P + \frac{\partial P}{\partial z}\cdot\frac{dz}{2}$$

$$dW = \gamma(dz)(dx)$$

<u>From ①</u>

$$\left(\cancel{P} - \frac{\partial P}{\partial x}\frac{dx}{2}\right)\cancel{(dz)} - \left(\cancel{P} + \frac{\partial P}{\partial x}\frac{dx}{2}\right)\cancel{(dz)} = 0$$

$$\boxed{\frac{\partial P}{\partial x} = 0}$$

$\boxed{\therefore \text{ Pressure does not vary in horizontal direction}}$

From ②

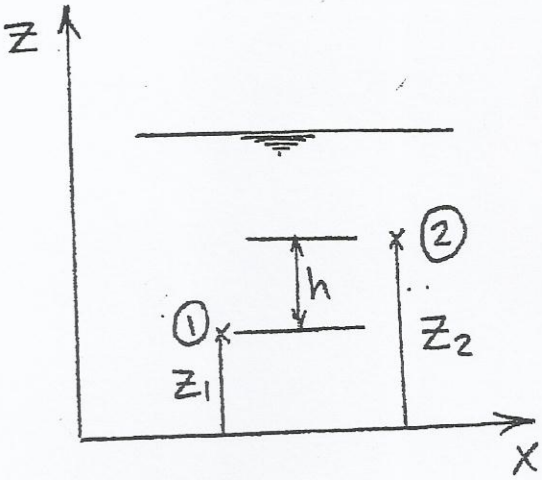
$$\left(\cancel{P}-\frac{\partial P}{\partial z}\cdot\frac{dz}{2}\right)\cancel{(dx)}-\left(\cancel{P}+\frac{\partial P}{\partial z}\cdot\frac{dz}{2}\right)\cancel{(dx)}-\gamma(dz)\cancel{(dx)}=0$$

$$-\frac{\partial P}{\partial z}\cancel{dz}-\gamma\cancel{dz}=0$$

$$\boxed{\frac{\partial P}{\partial z}=-\gamma}$$

Pressure Varies in Vertical direction

For two points ①, ②



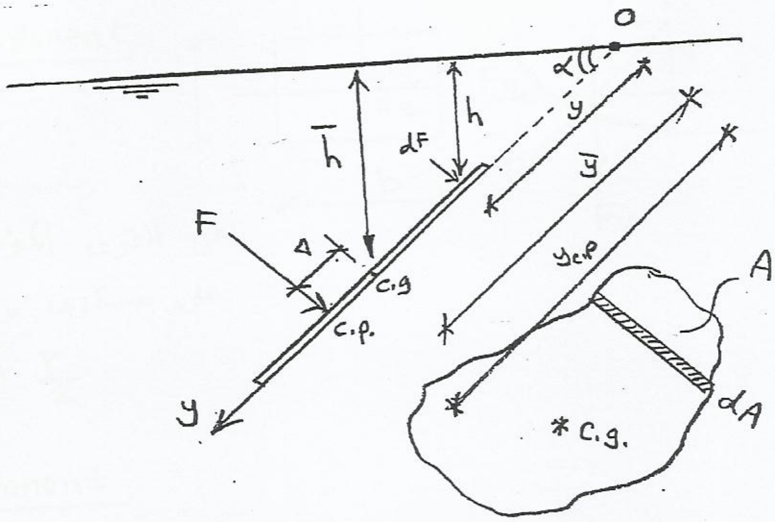
$$\int_{P_1}^{P_2} dP=-\gamma\int_{z_1}^{z_2} dz$$

$$\therefore P_2-P_1=-\gamma(z_2-z_1)=-\gamma h$$

$$\therefore \boxed{P_1=P_2+\gamma h}$$

12- For a submerged inclined plane surface, determine the resultant hydrostatic force due to the liquid acting on one side of the surface and its line of action

ⓐ Prove $F = \gamma A \bar{h}$, $\Delta = \dfrac{I_{c.g}}{A\bar{y}}$

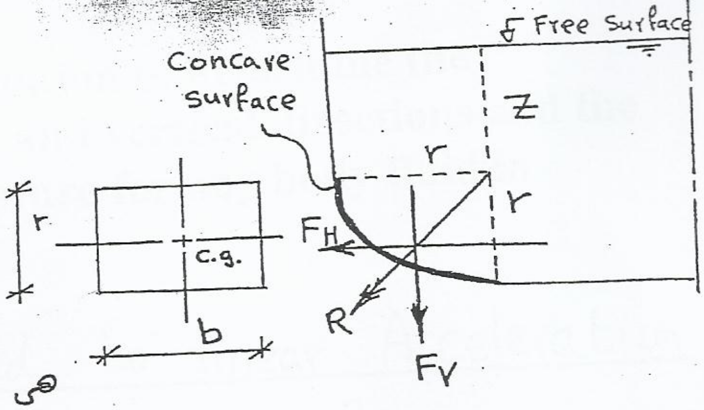


$$dF = p \, dA$$
$$= \gamma h \, dA$$
$$= \gamma y \sin\alpha \, dA$$
$$\int dF = \gamma \sin\alpha \int y \, dA$$
$$F = \gamma \sin\alpha \, A\bar{y}$$
$$\boxed{F = \gamma A \bar{h}}$$

$$dM = dF \cdot y$$
$$= (\gamma y \sin\alpha \, dA) \, y$$
$$\int dM = \gamma \sin\alpha \int y^2 \, dA$$
$$F . y_{c.p} = \gamma \sin\alpha \, I_o$$
$$\gamma \bar{y} \sin\alpha \, A \, y_{c.p} = \gamma \sin\alpha \, I_o$$
$$y_{c.p} = \frac{I_o}{A\bar{y}}$$
$$= \frac{I_{c.g} + A\bar{y}^2}{A\bar{y}} = \underbrace{\frac{I_{c.g}}{A\bar{y}}}_{\Delta} + \bar{y}$$
$$\therefore \boxed{\Delta = \frac{I_{c.g}}{A\bar{y}}}$$

13- Explain clearly how do you calculate the resultant force acting on a concave cylindrical surface submerged in a liquid

F_H : Horizontal Component



$$\boxed{F_H = \gamma A \bar{h}}$$

هى القوى المؤثرة على مسقط الـ Curved surface على مستوى رأسى

$$A = b r \quad , \quad \bar{h} = Z + \frac{r}{2}$$

F_V : Vertical Component

$$\boxed{F_V = \gamma \forall}$$

هى عبارة عن وزن السائل المحصور بين الـ Curved surface ومسقطه على الـ Free surface

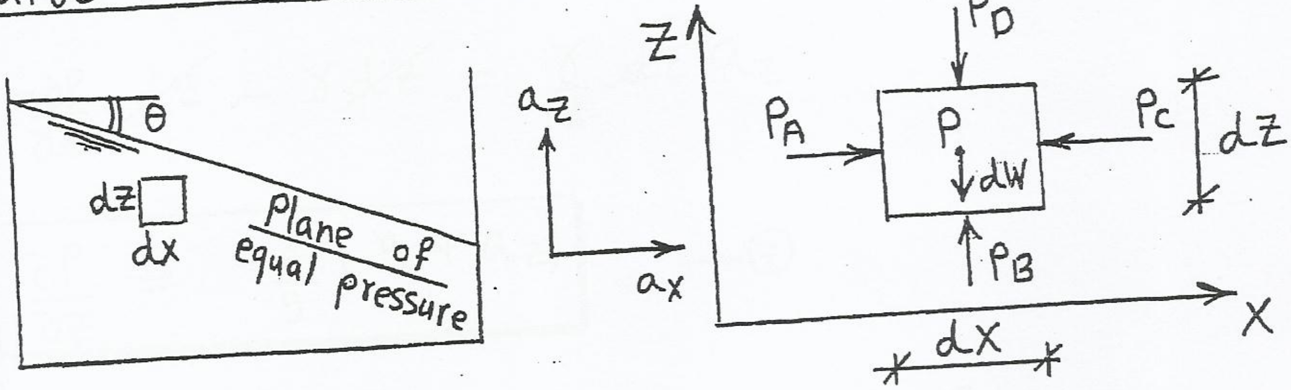
$$\forall = (Z + r)\, r\, b$$

R : Resultant

$$\boxed{R = \sqrt{F_H^2 + F_V^2}}$$

14- Apply the basic hydrostatics equation to determine the pressure variation in the horizontal and vertical directions and the slope of the surface of constant pressure for any body fluid in rigid body motion.

Fluid Masses Subjected to linear Acceleration



Consider a Small fluid element with dimensions (dxdz)

$$\sum F_x = P_A \, dz - P_c \, dz \quad \cdots\cdots \text{1}$$

$$\sum F_z = P_B \, dx - P_D \, dx - dW \quad \cdots\cdots \text{2}$$

$$\left.\begin{aligned} P_A &= P - \frac{\partial P}{\partial x}\frac{dx}{2} \quad , \quad P_c = P + \frac{\partial P}{\partial x}\cdot\frac{dx}{2} \\ P_B &= P - \frac{\partial P}{\partial z}\frac{dz}{2} \quad , \quad P_D = P + \frac{\partial P}{\partial z}\cdot\frac{dz}{2} \end{aligned}\right\} \rightarrow \text{3}$$

$$\sum F_x = dM \, a_x \quad \rightarrow \text{4}$$

$$P_A \, dz - P_c \, dz = dM \, a_x$$

$$\left(P - \frac{\partial P}{\partial x}\cdot\frac{dx}{2}\right)\cancel{dz} - \left(P + \frac{\partial P}{\partial x}\cdot\frac{dx}{2}\right)\cancel{dz} = \frac{\gamma}{g}\, dx \, \cancel{dz} \, a_x$$

$$-\frac{\partial P}{\partial x}\cancel{dx} = \frac{\gamma}{g}\cancel{dx}\, a_x$$

$$\boxed{\frac{\partial P}{\partial x} = -\frac{\gamma}{g}\, a_x} \quad \rightarrow \text{5}$$

$$\Sigma F_z = dM\, a_z \rightarrow (6)$$

$$P_B\, dx - P_D\, dx - dW = dM\, a_z$$

$$\left(P - \frac{\partial P}{\partial z}\frac{dz}{2}\right)\cancel{dx} - \left(P + \frac{\partial P}{\partial z}\cdot\frac{dz}{2}\right)\cancel{dx} - \gamma\,\cancel{dx}\,dz = \frac{\gamma}{g}\,\cancel{dx}\,dz\,a_z$$

$$-\frac{\partial P}{\partial z}\cdot\cancel{dz} - \gamma\,\cancel{dz} = \frac{\gamma}{g}\,\cancel{dz}\,a_z$$

$$\boxed{\frac{\partial P}{\partial z} = -\frac{\gamma}{g}\left(g + a_z\right)} \rightarrow (7)$$

$$dP = \frac{\partial P}{\partial x}\,dx + \frac{\partial P}{\partial z}\,dz = 0 \rightarrow (8)$$

$$\frac{dz}{dx} = \frac{-\partial P/\partial x}{\partial P/\partial z}$$

$$\boxed{\tan\theta = \frac{dz}{dx} = \frac{-a_x}{g \pm a_z}} \rightarrow (9)$$

15- State Archimedes Principle

Archimedes principle

Any weight, floating or submerged in a liquid, is acted upon by a ***buoyant force*** equal to the weight of the liquid displaced, and acts through the center of gravity of the displaced liquid.

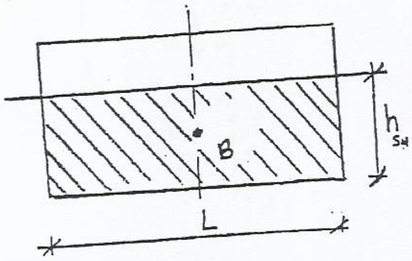
$\therefore$ $\boxed{\text{Weight of floating body} = \text{Weight of liquid displaced}}$

16- Define the following:-

a) Center of Buoyancy b) Metecenter c) Metacentric Height

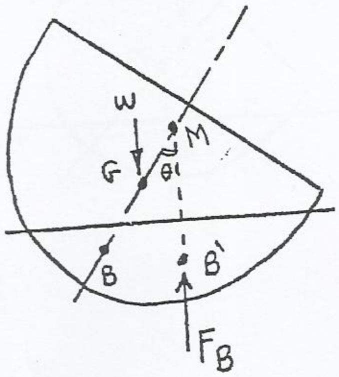
Center of Buoyancy

- It is the point of application of the force of buoyancy on the body.
- It is always the center of gravity of the volume of fluid displaced



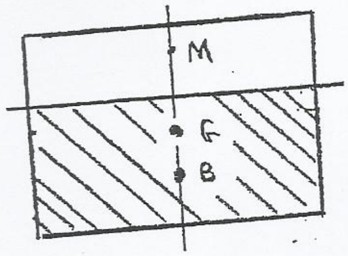
Metacentre

- The metacentre is the point of intersection of the axis of the body passing through the center of gravity (G) with the original centre of buoyancy (B) and a vertical line passing through the centre of buoyancy (B') of the tilted position of the body.
- The position of metacentre (M) remains practically constant for the small angle of tilt θ.



Metacentric Height:

- It is the distance between the centre of gravity of a floating body and the metacentre.
- $GM = BM - BG$

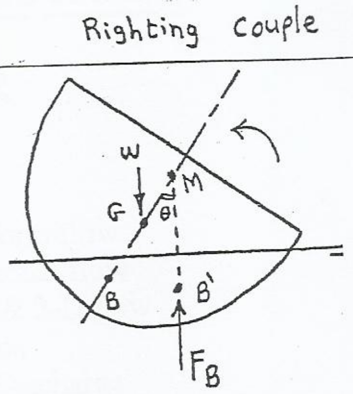


17- State the types of Equilibrium of floating bodies. Define each one using a neat sketch

Types of equilibrium of Floating bodies

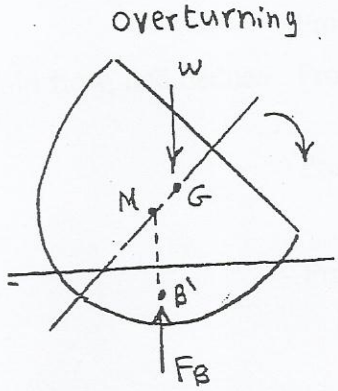
1. Stable equilibrium,
2. Unstable equilibrium and
3. Neutral equilibrium.

Stable Equilibrium



- It occurs when a body is tilted slightly by some external force, and then it returns back to its original position due to the weight and the upthrust.
- The position of metacentre M is higher than the center of gravity G.

Unstable Equilibrium



- It occurs when a body does not return to its original position from the slightly displaced angular position.
- The position of metacentre M is lower than G.

Neutral Equilibrium

- It occurs when a body, when given a small angular displacement, occupies a new position and remains at rest.
- The position of metacentre M coincides with G.

B

Theoretical Questions

Dynamics

1. Define the following expressions
   a) Steady flow & Unsteady flow — b)Uniform & Nonuniform flow
   c) Laminar flow & Turblent flow — d)Rotational & Irrotational flow
   e) Compressible & incompressible flow — f)1 Dimensional flow & 2-D flow
   g) Streamlines & Streamtubes. — h) Ideal and Real Fluids
   i) Viscous and inviscid flow — j) Mean velocity and Discharge

2. For a liquid in motion, what is meant by
   i) Potential head ii) Pressure head iii) Velocity head iv) Total head

3. Derive the equation of continuity for an incompressible fluid. Prove

4. Derive the Euler's equation of motion along a streamline for ideal fluid flow, and deduce from that Bernoulli's equation. Prove

5. Derive the equation of motion along a streamline for real fluid flow. Prove

6. Explain the theory of the Pitot tube.

7. What is the Venturimeter? Explain its working principle. (Show sketches and derive equations). Prove هام

8. What is meant by cavitation? Explain and give examples.

9. What is meant by i) Vena-Contracta ii) Vapour pressure

10. Derive a formula to calculate the coefficient of velocity of water jet passing through an orifice Prove

11. Derive a formula for the discharge through
    a) Large orifice b) Submerged orifice e) Partially submerged orifice

12. What is meant by a mouthpiece? Prove
    Derive an equation to calculate the headloss through a mouthpiece.

13. Derive an expression for the time required for emptying a tank of constant cross sectional area A and liquid height H through an orifice in the side of the tank of cross sectional area a and coefficient of discharge Cd. Prove

14. Obtain a formula for the discharge over a rectangular notch. Prove

15. Obtain a formula for the discharge over a triangular notch. What does this become if the angle of the notch is 90 Prove

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16. Derive the Impulse - momentum equation in the z- and x- directions.

17. Develop an expression for the force exerted by a jet of liquid, which strikes
    normally a stationary flat plate
    b) a fixed curved plate
    c) normally a moving plate
    d) normally a moving plate in the opposite direction
    e) a moving inclined vane

18. Sketch the velocity distribution for a fluid moving in a pipe in case of
    i) Laminar flow ii) Turblent flow

19. Sketch Moody Chart

20. Derive an expression for head loss due to friction in pipe for laminar flow Prove

21. Derive an expression for velocity distribution for viscous flow Prove

22. Derive an expression for the shear stress in pipe flow Prove

1- Define the following expressions

a) Steady flow & Unsteady flow
b) Uniform & Nonuniform flow
c) Laminar flow & Turblent flow
d) Rotational & Irrotational flow
e) Compressible & incompressible flow
f) 1 Dimensional flow & 2-D flow
g) Streamlines & Streamtubes.
h) Ideal and Real Fluids
i) Viscous and inviscid flow
j) Mean velocity and Discharge

1- Steady and unsteady flow

a- Steady flow

It occurs when velocity, acceleration,.. etc doesn't change with time

$$\frac{dV}{dt} = 0$$

b- Unsteady flow

It occurs when velocity or acceleration,.. etc changes with time

e.g. flow in a pipe whose valve is opening or closing

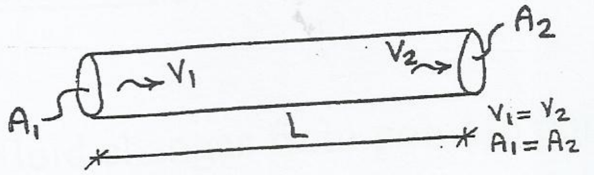
$$\frac{dV}{dt} \neq 0$$

2-Uniform and Non-uniform flow

a- Uniform flow

It occurs when velocity and cross-section remains constant over a given length

$$\frac{dV}{dL} = 0 \,, \quad \frac{dA}{dL} = 0$$



b- Non-uniform flow

It occurs when velocity or cross-section changes over a given length

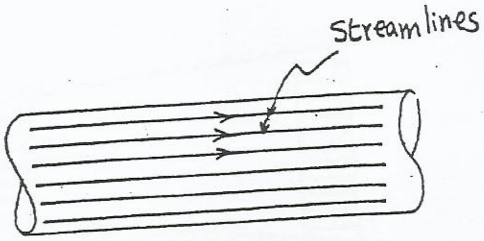
$$\frac{dV}{dL} \neq 0 \,, \quad \frac{dA}{dL} \neq 0$$

3- Laminar and turbulent flow

a- Laminar flow

It occurs when fluid particles in parallel paths and do not intersect

e.g. flow through capillary tubes, ground water, and blood in veins.

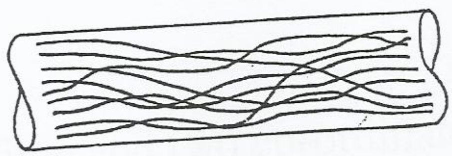


$R_n < 2000$

b- Turblent flow

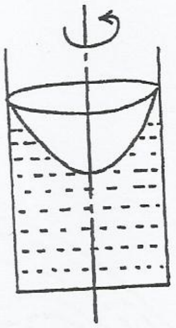
It occurs when fluid particles move in random motion

e.g. Nearly in all flow in pipes



$R_n > 4000$

4- Rotational and Irrotational flow



a- Rotational flow

It occurs when fluid particles have a rotation about an axis

b- Irrotational flow

It occurs when fluid particles don't have a rotation about an axis

5- Compressible and incompressible flow

a- Compressible flow

It occurs when the density of the fluid changes from point to point

e.g. Flow of gases through orifices and nozzles

b- Incompressible flow

It occurs when the density is constant for fluid flow

e.g. Liquids are generally considered flowing incompressibly

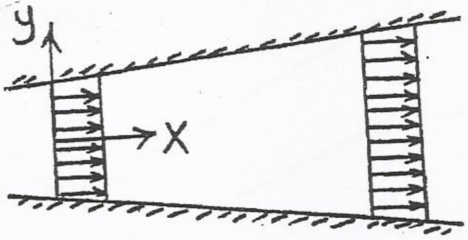
6- One, two three dimensional flow

a- One dimensional flow

It occurs when the velocity is a function of time and one co-ordinate.

$v = f(x, t)$

e.g. Flow through a straight uniform diameter pipe



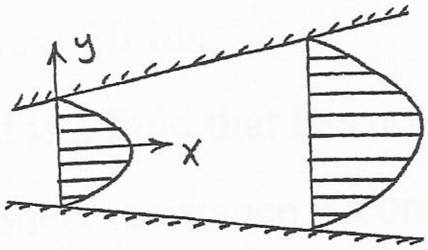
The flow is never truly 1 dimensional, because viscosity causes the fluid velocity to be zero at the boundaries.

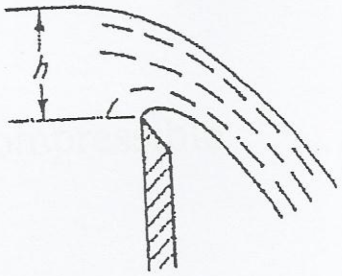
b- Two dimensional flow

It occurs when the velocity is a function of time and two co-ordinates

$v = f(x, y, t)$

e.g. Flow in the main stream of a wide river



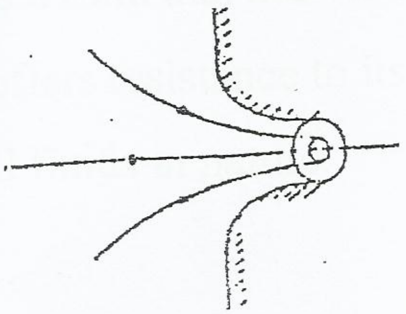


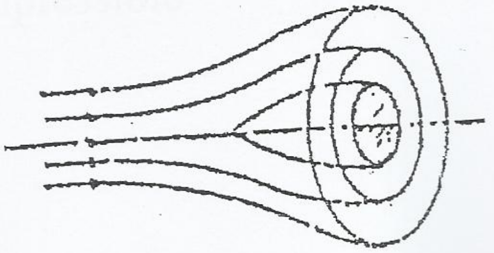
c- Three dimensional flow

It occurs when the velocity is a function of time and three co-ordinates

$v = f(x, y, z, t)$

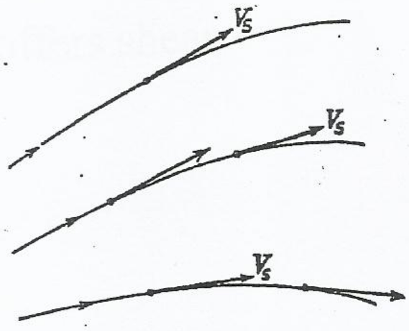
e.g. Flow in a converging or diverging pipe





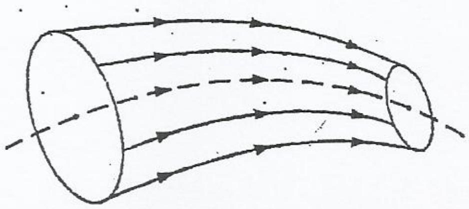
7- Stream lines and streamtubes

a- Streamlines



- Streamlines are imaginary curves drawn to show the direction of fluid flow
- The tangent at any point gives the velocity direction

b- Streamtubes



- A stream tube is a fluid mass bounded by a group of streamlines

8- Ideal and Real Fluids

a- Ideal Fluids

- It is a fluid that has no viscosity, and incompressible
- Shear resistance is considered zero
- Ideal fluid does not exist in nature

e.g. Water and air are assumed ideal

b- Real Fluids

- It is a fluid that has viscosity, and compressible
- It offers resistance to its flow

e.g. All fluids in nature

9- Viscous and inviscid flow

a- Viscous flow

- It occurs for fluids that have viscosity which offers shear resistance to the flow
- A part of the total energy is lost in flow

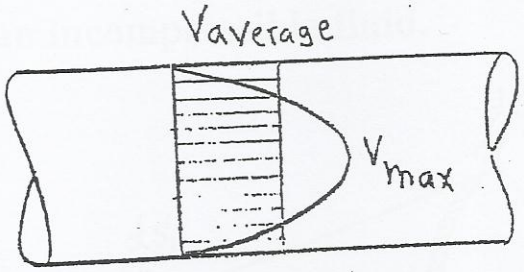
b- Inviscid flow

- It occurs for fluids that have no viscosity
- No shear resistance to the flow
- The total energy remains constant.

10- Mean velocity and Discharge

a- Mean velocity

It is the average velocity passing a given section

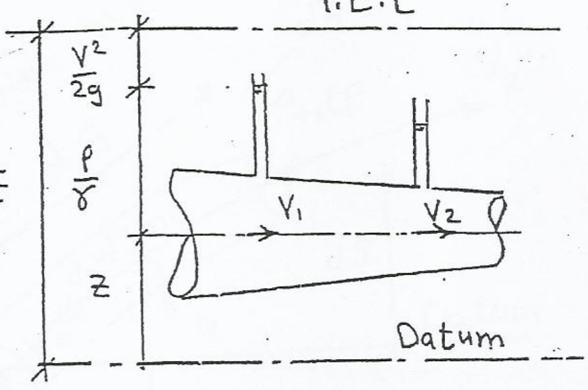


b- Discharge

It is the rate of Volume of liquid passing a given cross-section

2- For a liquid in motion, what is meant by

i) Potential head ii) Pressure head iii) Velocity head iv) Total head



a) Potential head $= \frac{\text{Potential energy}}{\text{unit weight}}$

$= \frac{mg\,Z}{mg} = \boxed{Z}$

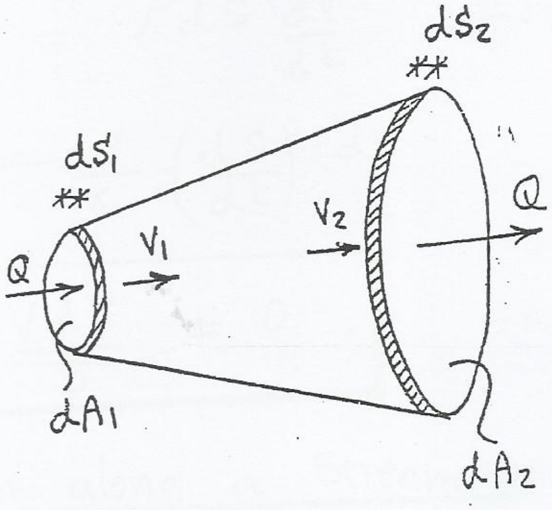
b) Pressure head $= \frac{\text{Pressure energy}}{\text{unit weight}}$

$= \boxed{\frac{P}{\gamma}}$

c) Velocity head $= \frac{\text{Kinetic energy}}{\text{unit weight}} = \boxed{\frac{V^2}{2g}}$

d) Total head $=$ $\boxed{E = \frac{P}{\gamma} + \frac{V^2}{2g} + Z}$

3- Derive the equation of continuity for an incompressible fluid.



$$dM_1 = dM_2$$

$$\rho_1\, dA_1 \frac{ds_1}{dt} = \rho_2\, dA_2 \frac{ds_2}{dt}$$

$$\rho_1\, dA_1\, V_1 = \rho_2\, dA_2\, V_2$$

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

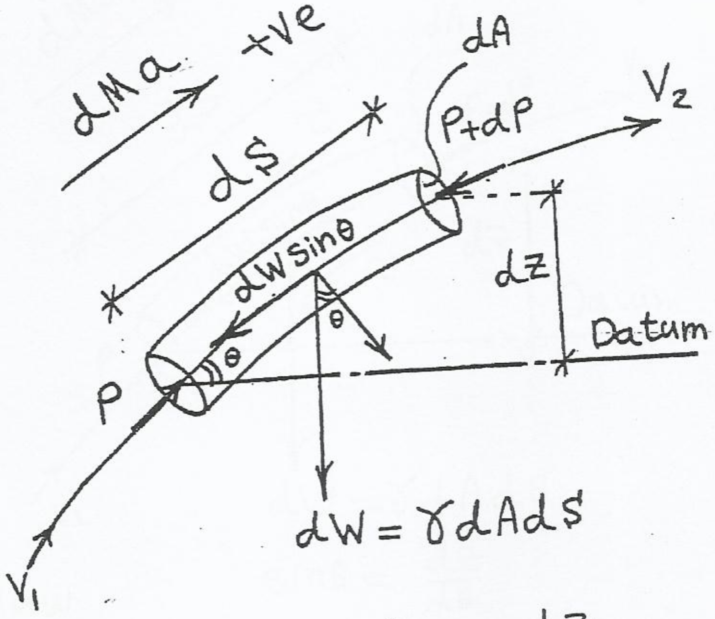
For incompressible fluids. $\rho_1 = \rho_2$

$$\boxed{A_1 V_1 = A_2 V_2}$$

4- Derive the Euler's equation of motion along a streamline for ideal fluid flow, and deduce from that Bernoulli's equation.

Ideal Fluid

Euler & Bernoulli's eqn



Consider a fluid element of cross-section dA and length dS moving along a streamline.

$$dW = \gamma\, dA\, dS$$

$$\sin\theta = \frac{dz}{dS}$$

Applying Newton 2nd law

$$P\,dA - (P + dP)\,dA - dW \sin\theta = dM\,a$$

$$\cancel{P\,dA} - \cancel{P\,dA} - dP\,\cancel{dA} - \gamma \cancel{dA}\cancel{dS}\left(\frac{dz}{\cancel{dS}}\right) = \rho \cancel{dA}\, dS\, \frac{dV}{dt}$$

$$-dP - \gamma\, dz = \rho\, dS\, \frac{dV}{dt} \qquad \div \gamma$$

$$\frac{-dP}{\gamma} - dz - \frac{\rho}{\gamma}\left(\frac{dS}{dt}\right) dV$$

Euler's eqn $\boxed{\dfrac{dP}{\gamma} + dz + \dfrac{V\,dV}{g} = 0}$

Equation of Steady motion along a Streamline

By integration of Euler's equation

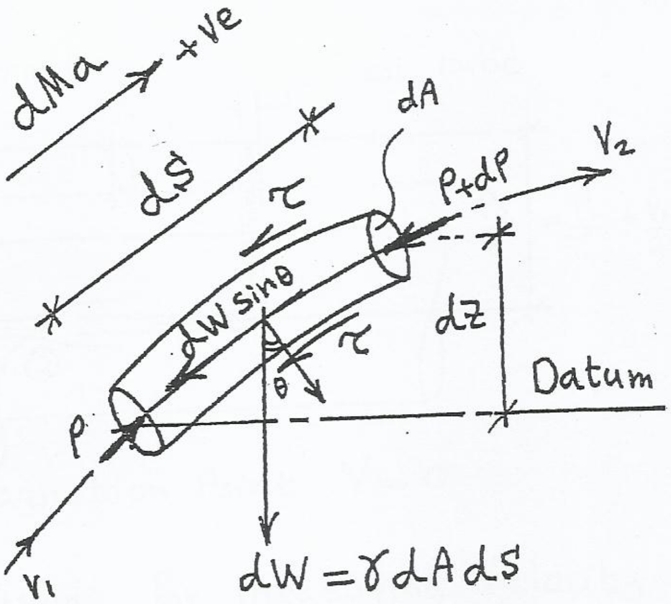
Bernoulli's eqn $\boxed{\dfrac{P}{\gamma} + Z + \dfrac{V^2}{2g} = \text{Constant}}$

Pressure head + Position head + Velocity head = Total head

5- Derive the equation of motion along a streamline for real fluid flow.

Real Fluid

Real fluid has an additional force acting caused by Friction



$F = \tau\, dA = \tau(2\pi r)ds$

$dW = \gamma\, dA\, ds$

Applying Newton 2nd law

$\sin\theta = \frac{dz}{ds}$

$$P dA - (P + dP)dA - dW\sin\theta - \tau(2\pi r)ds = dMa$$

$$-dP\,\cancel{dA} - \gamma\,\cancel{dA\,ds}\left(\frac{dz}{\cancel{ds}}\right) - \tau\left(2\frac{\cancel{dA}}{r}\right)ds = \rho\,\cancel{dA}\,ds\,\frac{dV}{dt}$$

$\because dA = \pi r^2 \qquad \div \gamma$

$$\frac{-dP}{\gamma} - dz - \frac{2\tau\, ds}{\gamma r} = \frac{V dV}{g}$$

$$\frac{dP}{\gamma} + dz + d\left(\frac{V^2}{2g}\right) = \frac{-2\tau\, ds}{\gamma r}$$

$$\int_1^2 \frac{dP}{\gamma} + \int_1^2 dz + \int_1^2 d\left(\frac{V^2}{2g}\right) = \int_1^2 \frac{-2\tau\, ds}{\gamma r}$$

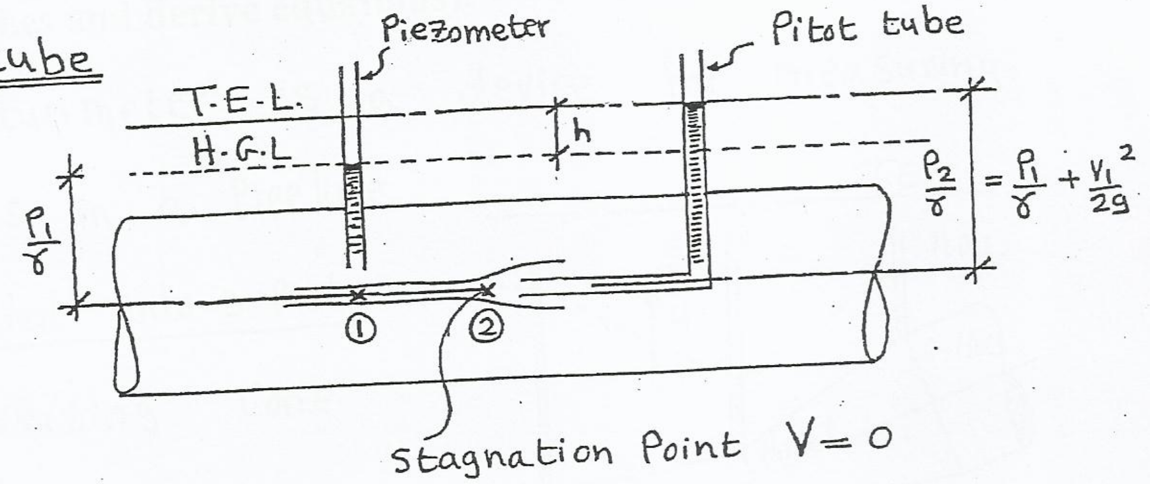
$$\boxed{\left(\frac{P_1}{\gamma} + Z_1 + \frac{V_1^2}{2g}\right)_1 - \frac{2\tau L}{\gamma r} = \left(\frac{P_2}{\gamma} + Z_2 + \frac{V_2^2}{2g}\right)_2}$$

Dims of $\frac{2\tau L}{\gamma r} = \frac{N/m^2 * m}{N/m^3 * m} = m$

$$\boxed{H_1\,(\text{Total energy at Sec 1}) - h_L\,(\text{head lost})_{1 \to 2} = H_2\,(\text{T.E. at Sec 2})}$$

6- Explain the theory of the Pitot tube.

Pitot tube



* The Pitot tube is used for measuring Velocity head of a flowing liquid.
* It is Placed with its opening in the direction of flow
* The liquid flows up the tube until all its Kinetic Energy is Converted to Potentral Energy

Applying Bernoullis eqn between 1, 2

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + \cancel{Z_1} = \frac{P_2}{\gamma} + \underbrace{\cancel{\frac{V_2^2}{2g}}}_{=0} + \cancel{Z_2}$$

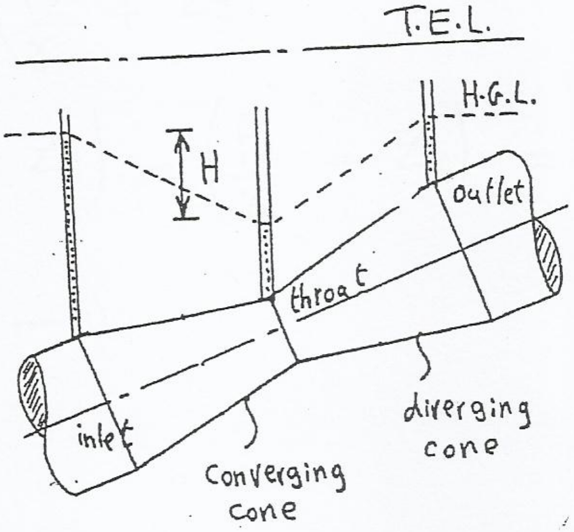
$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} = \frac{P_2}{\gamma}$$

$$\frac{P_2 - P_1}{\gamma} = \frac{V_1^2}{2g}$$

$$\boxed{h = \frac{V_1^2}{2g}}$$

7- What is the Venturimeter? Explain its working principle.
(Show sketches and derive equations).

The Venturimeter is a device for measuring discharges in a Pipe line



It is divided into 3 Parts

1- Converging Cone
2- Throat
3- Diverging Cone

Its Working Principle

The Converging Cone Converts Pressure energy to Kinetic energy, So the Velocity at the throat increases and the hydraulic gradient line decreases

The change in the Piezometric head Can be measured

$$\boxed{H = \left(\frac{P_1}{\gamma} + Z_1\right) - \left(\frac{P_2}{\gamma} + Z_2\right)}$$

and the discharge Can be Calculated

$$\boxed{Q = \frac{A_1 A_2 \sqrt{2gH}}{\sqrt{A_1^2 - A_2^2}}}$$

The diverging Cone Converts Kinetic energy into Pressure energy

Applying Bernoulli equation between ①, ②

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$

Let $H = \left(\frac{P_1}{\gamma} + Z_1\right) - \left(\frac{P_2}{\gamma} + Z_2\right)$

$$\frac{V_2^2}{2g} - \frac{V_1^2}{2g} = \left(\frac{P_1}{\gamma} + Z_1\right) - \left(\frac{P_2}{\gamma} + Z_2\right) = H$$

$$V_2^2 - V_1^2 = 2gH \quad \rightarrow ①$$

$\because A_1 V_1 = A_2 V_2$

$$\therefore V_1 = V_2 \frac{A_2}{A_1} \quad \rightarrow ②$$

$$V_2^2 - V_2^2 \frac{A_2^2}{A_1^2} = 2gH$$

$$V_2^2 \left[1 - \frac{A_2^2}{A_1^2}\right] = 2gH$$

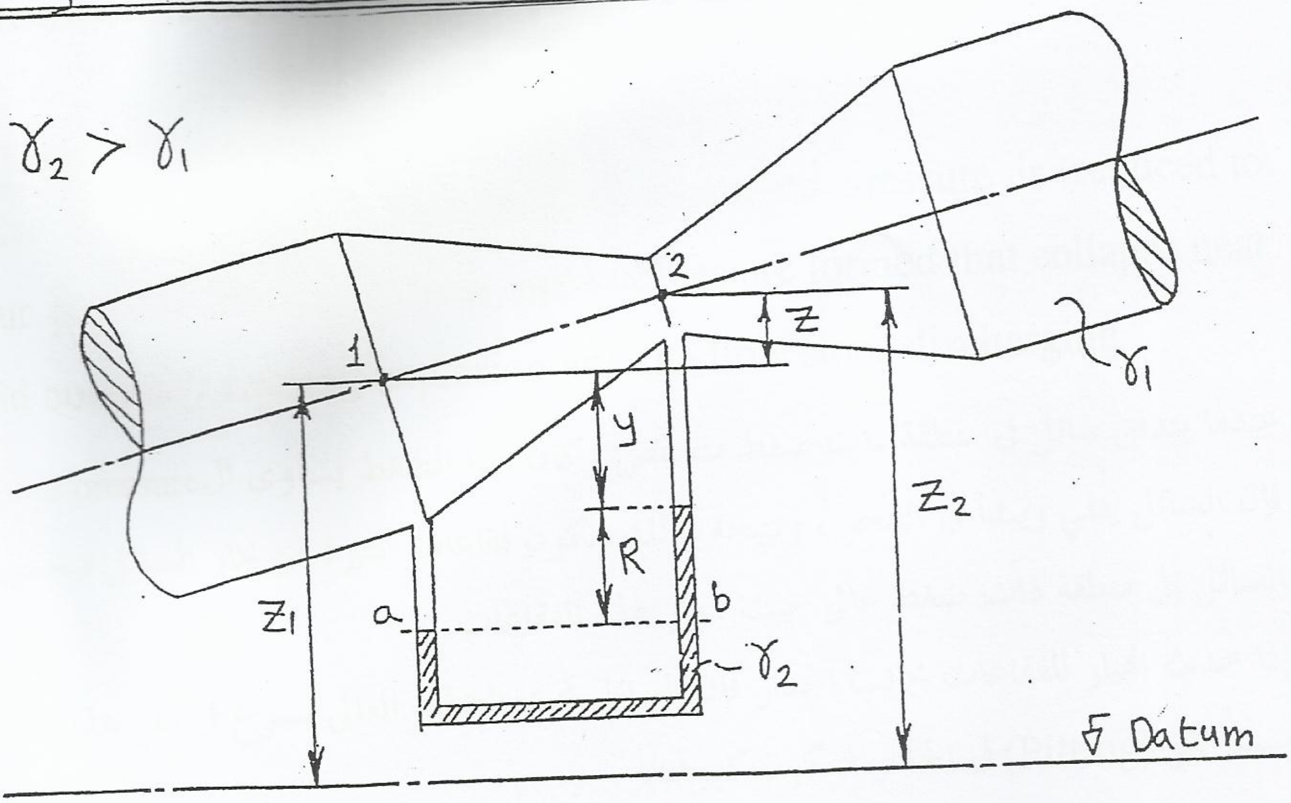
$$V_2^2 \left(\frac{A_1^2 - A_2^2}{A_1^2}\right) = 2gH$$

$$V_2 = \frac{A_1 \sqrt{2gH}}{\sqrt{A_1^2 - A_2^2}}$$

$$Q_{th} = V_2 A_2 = \frac{A_1 A_2 \sqrt{2gH}}{\sqrt{A_1^2 - A_2^2}}$$

$$\boxed{Q_{act} = \frac{C_d A_1 A_2 \sqrt{2gH}}{\sqrt{A_1^2 - A_2^2}}}$$

Using Manometer With Venturimeter



$$P_a = P_1 + \gamma_1 y + \gamma_1 R$$

$$P_b = P_2 + \gamma_1 Z + \gamma_1 y + \gamma_2 R$$

$$P_a = P_b$$

$$P_1 + \cancel{\gamma_1 y} + \gamma_1 R = P_2 + \gamma_1 Z + \cancel{\gamma_1 y} + \gamma_2 R$$

$$P_1 - P_2 - \gamma_1 Z = R(\gamma_2 - \gamma_1) \qquad \div \gamma_1$$

$$\frac{P_1}{\gamma_1} - \frac{P_2}{\gamma_1} - Z = R\left(\frac{\gamma_2}{\gamma_1} - 1\right)$$

$$\because Z = Z_2 - Z_1 \qquad \Rightarrow \qquad -Z = Z_1 - Z_2$$

$$\frac{P_1}{\gamma} - \frac{P_2}{\gamma} + Z_1 - Z_2 = R\left(\frac{\gamma_2}{\gamma_1} - 1\right)$$

$$\boxed{H = \left(\frac{P_1}{\gamma} + Z_1\right) - \left(\frac{P_2}{\gamma} + Z_2\right) = R\left(\frac{\gamma_2}{\gamma_1} - 1\right)}$$

8- What is meant by cavitation? Explain and give examples.

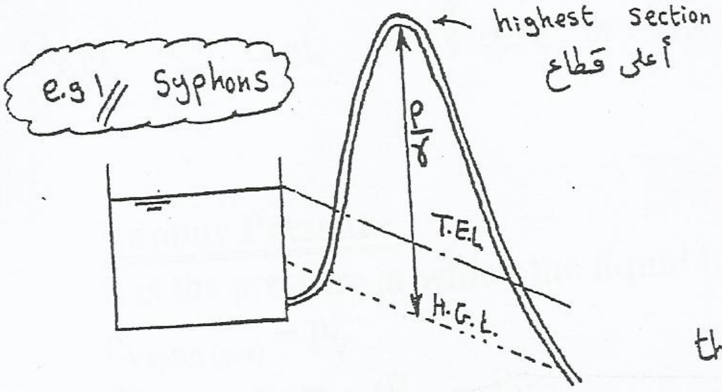
Cavitation التكهف

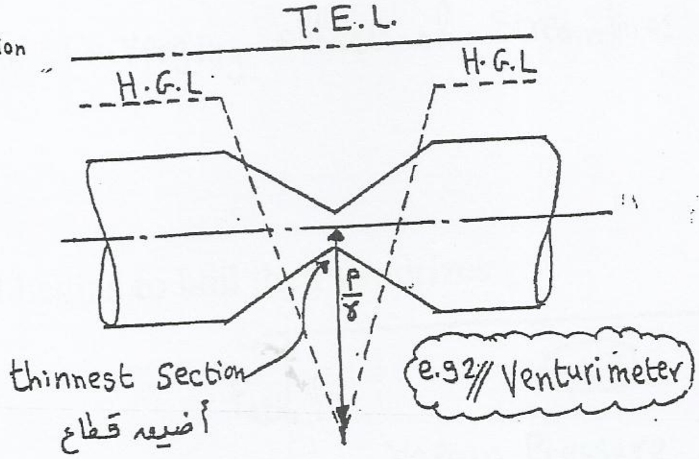
When a liquid flows into a region where its pressure is reduced to vapour pressure, the liquid boils and bubbles are formed that collapse near a solid boundary causing explosion and the flow stops discharging.

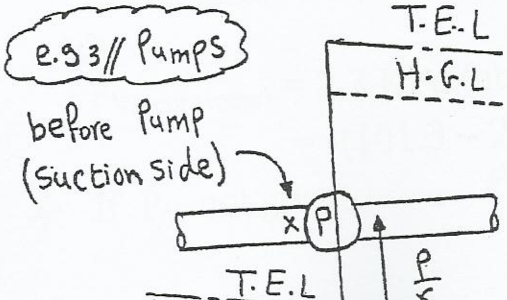
عندما يتدفق سائل فى منطقة ذات ضغط منخفض وكان هذا الضغط يساوى الـVapour pressure فإن السائل يغلي ويبدأ فى التبخر ، ونتيجة لذلك تتكون فقاعات كثيرة من بخار السائل فيحملها السائل إلى منطقة ذات ضغط عالى حيث تنهار هذه الفقاعات.

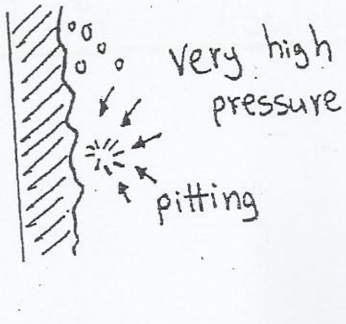
إذا حدث انهيار للفقاعات بجانب الجدار فإن السَائل تحت الضغط العالى يُسرع فى ملئ هذا الفراغ مسبباً نقر (Pitting) فى الجدار فتتكون كهوف.

هذا التتابع قد يسبب انهيار للجدار وبالتالى يقف السائل عن التدفق.







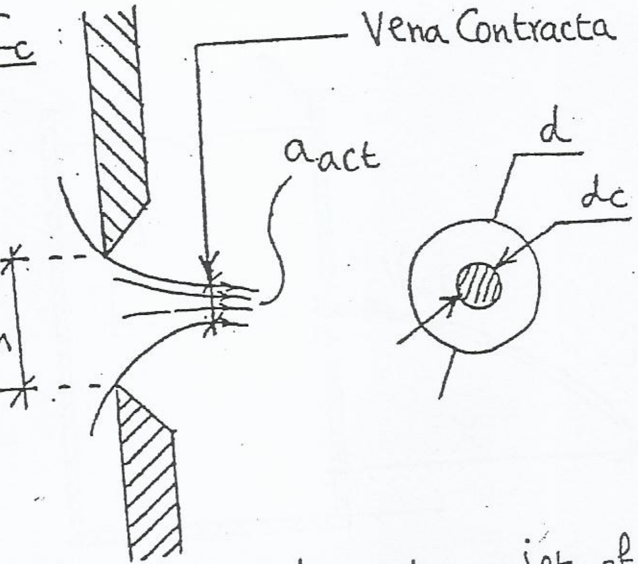


9- ~~What is meant by~~

i) Vena-Contracta

ii) Vapour pressure

<u>Coefficient of Contraction C_c</u> :



$$a_{th} = \frac{\pi d^2}{4}$$

a_{act} = area at the Vena Contracta

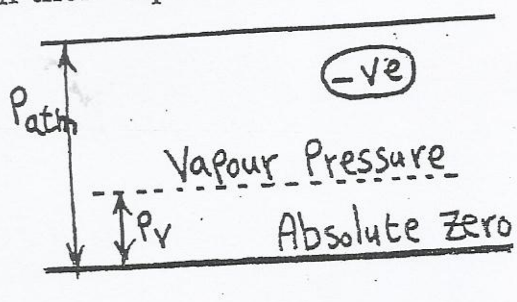
<u>Vena Contracta</u>

It is the Section at the minimum diameter jet of Water discharging from the Orifice at which the Stream-lines Starts to become parallel

$a_{act} < a_{th}$ ((due to the Converging effect of Streamlines

<u>Vapour Pressure</u>

It is the pressure at which the liquid begins to boil then vaporizes



$P_{Vapour\,(abs)} = P_V$

$P_{Vapour\,(gauge)} = -(P_{atm} - P_V)$

e.g

$P_{Vapour\,(water)} = 2.3$ KPa (abs)

$= -(101.3 - 2.3) = -99$ KPa (gauge)

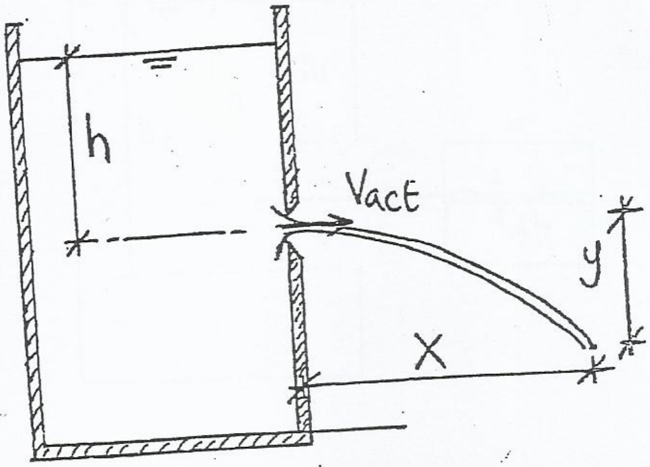
* If P_V not given $\Rightarrow P_V = -8\gamma_w$ N/m^2 (Pa)

10- Derive a formula to calculate the coefficient of velocity of water jet passing through an orifice

Applying Newton's equation

$$S = V_o t + \frac{1}{2} a t^2$$

$\because$ no acceleration in hzl direction



$\therefore a_x = 0$, $S = X$ لا توجد عجلة أفقية

$$X = V_{ox} t = V_{act} t$$

$$\boxed{t = \frac{X}{V_{act}}} \rightarrow (1)$$

$\because$ no initial velocity in Vl direction

$V_{oy} = 0$, $S = y$, $a_y = g$

$$\boxed{y = \frac{1}{2} g t^2} \rightarrow (2)$$

from (1), (2)

$$y = \frac{1}{2} g \frac{X^2}{V_{act}^2} \rightarrow (3)$$

$$\because V_{act} = C_v V_{th} = C_v \sqrt{2gh}$$

$$V_{act}^2 = C_v^2 (2gh) \rightarrow (4)$$

from (3), (4)

$$y = \frac{1}{2} \frac{g X^2}{C_v^2 (2gh}$$

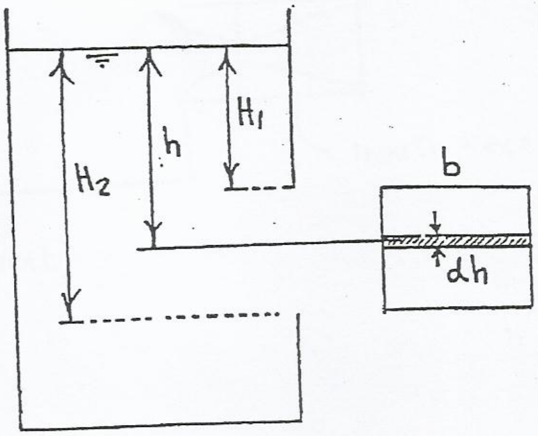
$$C_v^2 = \frac{X^2}{4yh} \quad \text{or} \quad \boxed{C_v = \frac{X}{2\sqrt{yh}}}$$

11- Derive a formula for the discharge through

a) Large orifice b) Submerged orifice e) Partially submerged orifice

Flow through Large Orifices

from Continuity equation



$$dQ = V dA$$

where ; $V = \sqrt{2gh}$

$dA = b\,dh$

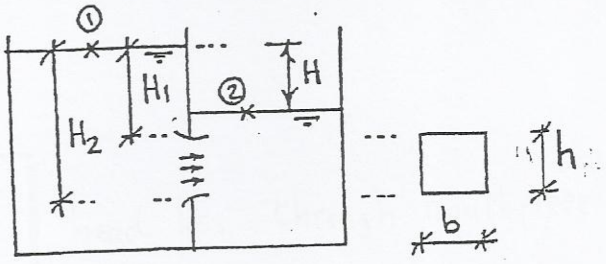
$$dQ = \sqrt{2gh}\, b\, dh$$

$$Q = b\sqrt{2g}\int_{H_1}^{H_2} h^{1/2}$$

$$Q = \frac{2}{3} b\sqrt{2g}\left(H_2^{3/2} - H_1^{3/2}\right)$$

$$\boxed{Q = \frac{2}{3} C_d\, b\sqrt{2g}\left(H_2^{3/2} - H_1^{3/2}\right)}$$

Disharge through Fully Submerged Large orifices (drowned)



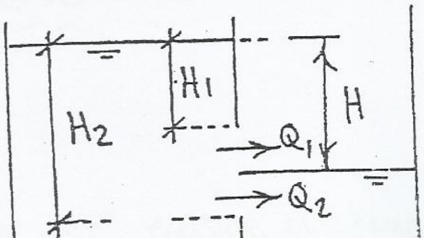
$$Q = AV$$

$$= bh\left(\sqrt{2gH}\right)$$

$$= b(H_2 - H_1) * \sqrt{2gH}$$

$$\boxed{Q_{act} = C_d\, b\,(H_2 - H_1) * \sqrt{2gH}}$$

Discharge through Partially Submerged Orifice



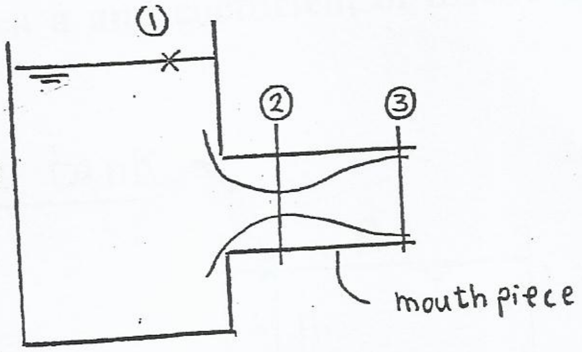
$$Q_1 = \frac{2}{3} C_d\, b\sqrt{2g}\left(H^{3/2} - H_1^{3/2}\right)$$

$$Q_2 = C_d\, b\,(H_2 - H) * \sqrt{2gH}$$

$$Q_{act} = Q_1 + Q_2$$

12- What is meant by a mouthpiece? Derive an equation to calculate the headloss through a mouthpiece.

Flow through mouthpieces



It is a small tube attached to an orifice

$h_{L_{1\to2}}$ neglected

$$\boxed{h_{L_{2\to3}} = \frac{(V_2 - V_3)^2}{2g}}$$ (sudden enlargment)

$$C_c = \frac{a_{act}}{a_{th}} = \frac{a_2}{a_3}$$

$$\because\ a_2 V_2 = a_3 V_3$$

$$C_c a_3 V_2 = a_3 V_3$$

$$\boxed{V_2 = \frac{V_3}{C_c}}$$

$$\therefore\ h_{L_{2\to3}} = \frac{\left(\frac{V_3}{C_c} - V_3\right)^2}{2g}$$

$$\boxed{h_{L_{2\to3}} = \left(\frac{1}{C_c} - 1\right)^2 \frac{V_3^2}{2g}}$$ head loss through mouthpiece

If we neglect h_L from ② → ③

and applying Bernoulli eqn. between ②,③

$$\frac{P_2}{\gamma} + \frac{V_2^2}{2g} + 0 = 0 + \frac{V_3^2}{2g} + 0$$

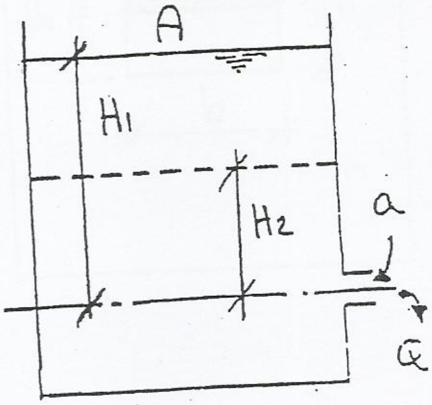
$$\frac{P_2}{\gamma} = \frac{V_3^2 - V_2^2}{2g} = -ve$$

due to -ve pressure, discharge increase.

Check Cavitation

13- Derive an expression for the time required for emptying a tank of constant cross sectional area A and liquid height H through an orifice in the side of the tank of cross sectional area a and coefficient of discharge Cd.

<u>Time required to empty the tank</u>



$$Q\,dt = -A\,dh$$

$$C_d\, a\sqrt{2gh}\, dt = -A\,dh$$

$$\int_0^{T} dt = \frac{-A}{C_d\, a\sqrt{2g}} \int_{H_1}^{H_2} h^{-\frac{1}{2}}\, dh$$

$$T = \frac{-2A}{C_d\, a\sqrt{2g}} \left(H_2^{\frac{1}{2}} - H_1^{\frac{1}{2}}\right)$$

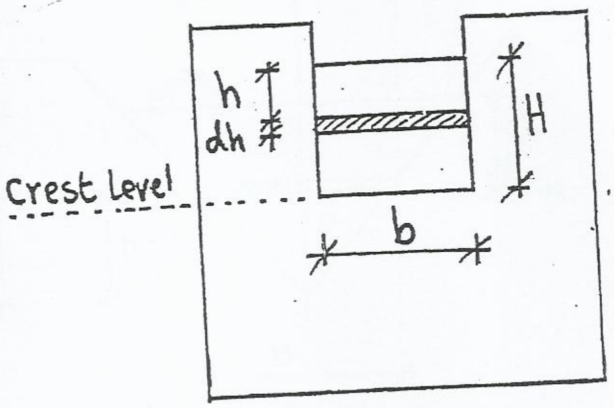
$$\boxed{T = \frac{2A}{C_d\, a\sqrt{2g}} \left(\sqrt{H_1} - \sqrt{H_2}\right)}$$

H_1 = initial head in tank

H_2 = final head in tank

If it is required to empty the tank $H_2 = 0$

14- Obtain a formula for the discharge over a rectangular notch.



From Continuity equation

$$dQ = V\,dA$$

where; $V = \sqrt{2gh}$

$dA = b\,dh$

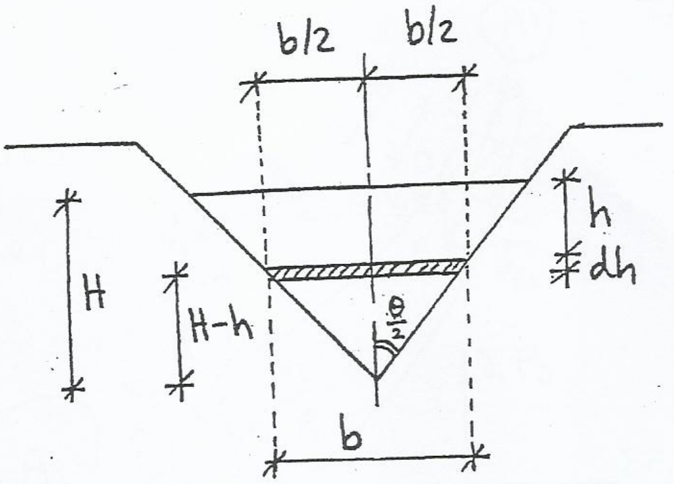
$$dQ = \sqrt{2gh}\; b\,dh$$

$$Q = b\sqrt{2g}\int_0^{H} h^{\frac{1}{2}}\,dh$$

$$Q = \frac{2}{3}\, b\sqrt{2g}\; H^{3/2}$$

$$\boxed{Q_{act} = \frac{2}{3}\, C_d\, b\sqrt{2g}\; H^{3/2}}$$

15- Obtain a formula for the discharge over a triangular notch. What does this become if the angle of the notch is 90°



From Continuity equation

$$dQ = V\,dA$$

where; $V = \sqrt{2gh}$

$$dA = b\,dh$$

$$b = 2(H-h)\tan\frac{\theta}{2}$$

$$\therefore dQ = \sqrt{2gh}\left[2(H-h)\tan\frac{\theta}{2}\right]dh$$

$$Q = 2\sqrt{2g}\,\tan\frac{\theta}{2}\int_0^H (H-h)\,h^{1/2}\,dh$$

$$\int_0^H \left(H h^{1/2} - h^{3/2}\right) dh$$

$$\left| \frac{2}{3} H h^{3/2} - \frac{2}{5} h^{5/2} \right|_0^H$$

$$Q = 2\sqrt{2g}\,\tan\frac{\theta}{2}\left(\frac{2}{3}H^{5/2} - \frac{2}{5}H^{5/2}\right)$$

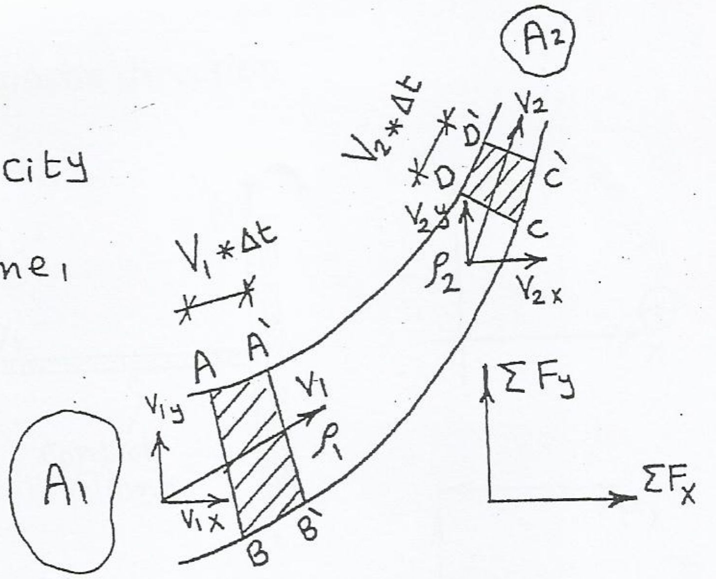
$$Q = \frac{8}{15}\sqrt{2g}\,\tan\frac{\theta}{2}\,H^{5/2}$$

$$\boxed{Q_{act} = \frac{8}{15}C_d\sqrt{2g}\,\tan\frac{\theta}{2}\,H^{5/2}}$$

at $\theta = 90^\circ \Rightarrow \tan\frac{90}{2} = 1$

$$Q_{act} = \frac{8}{15}C_d\sqrt{2g}\,H^{5/2}$$

16- Derive the Impulse - momentum equation in ~~the z- and x- directions.~~



$$Momentum = Mass * Velocity$$

$$Mass_{AA`BB`} = \rho_1 * Volume_1$$

$$M_{AA`BB`} = \rho_1 A_1 (V_1 \Delta t)$$

$$M_{CC`DD`} = \rho_2 A_2 (V_2 \Delta t)$$

<u>X- direction</u>

$$\text{Change in Momentum in X direction} = M_{CC`DD`} * V_{2x} - M_{AA`BB`} * V_{1x}$$

$$\Sigma F_x t = M_{CC`DD`} V_{2x} - M_{AA`BB`} V_{1x}$$

$$\Sigma F_x = \frac{(\rho_2 A_2 V_2 \Delta t) V_{2x} - (\rho_1 A_1 V_1 \Delta t) V_{1x}}{\Delta t}$$

$$\Sigma F_x = \rho_2 A_2 V_2 V_{2x} - \rho_1 A_1 V_1 V_{1x}$$

For incompressible Fluid $\rho_1 = \rho_2 = \rho$

For Steady State Condition $A_1 V_1 = A_2 V_2 = Q$

$$\boxed{\Sigma F_x = \rho Q (V_2 - V_1)_x}$$

<u>Similarly in y direction</u>

$$\Sigma F_y = \rho Q (V_2 - V_1)_y$$

<u>In any direction</u>

$$\boxed{\Sigma F = \rho Q [V_{final} - V_{initial}]}$$

17- Develop an expression for the force exerted by a jet of liquid, which strikes

a) normally a stationary flat plate

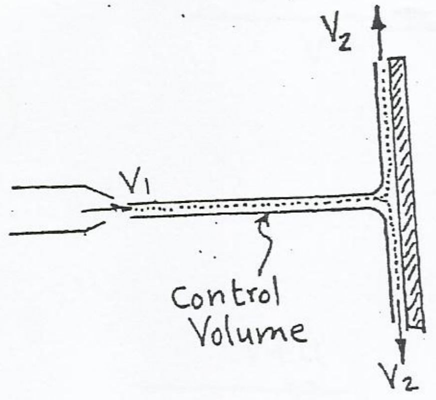
b) a fixed curved plate

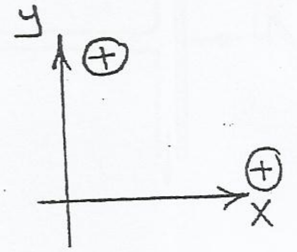
c) normally a moving plate

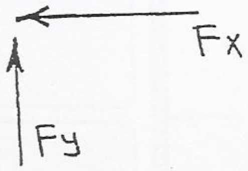
d) normally a moving plate in the opposite direction

e) a moving inclined vane

(a) Let F_x, F_y are Force Components on Water







Momentum equation in X-direction

$$-F_x = (0) - (\rho Q V_1)$$

$$\therefore \boxed{F_x = \rho Q V_1}$$

Momentum equation in y-direction

$$+F_y = (\rho Q V_2 - \rho Q V_2) - (0)$$

$$\boxed{F_y = 0}$$

(b) Force on a fixed curved Plate

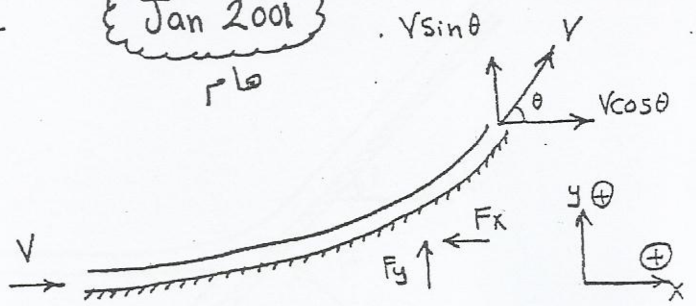
Jan 2001

طلع

$$\Sigma F_x = \rho Q (V_f - V_i)$$

$$-F_x = \rho Q (V\cos\theta - V)$$

$$F_x = \rho Q V (1 - \cos\theta)$$



$$+F_y = \rho Q (V\sin\theta - 0)$$

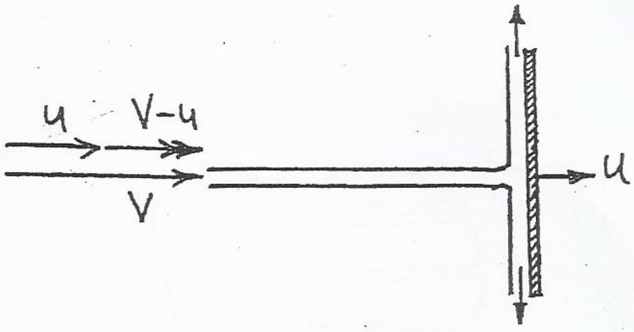
$$F_y = \rho Q V \sin\theta$$

Moving Plates

ⓒ

$$F_x = \rho Q(V-u)$$

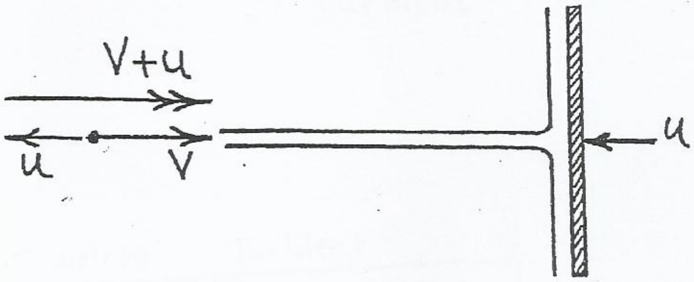
where $Q = A_{jet}(V-u)$



ⓓ

$$F_x = \rho Q(V+u)$$

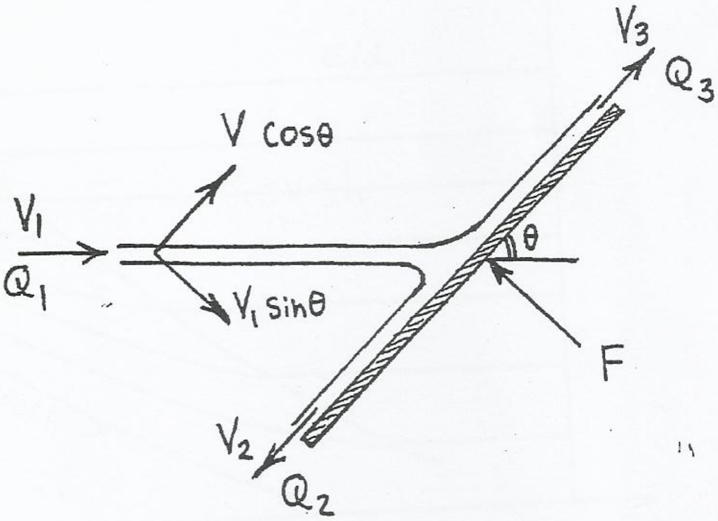
where $Q = A_{jet}(V+u)$



Solving ⊥ to the Plate

$$\Sigma F = \rho Q(\overset{0}{\cancel{V_f}} - V_I)$$

$$F = \rho Q V_1 \sin\theta$$

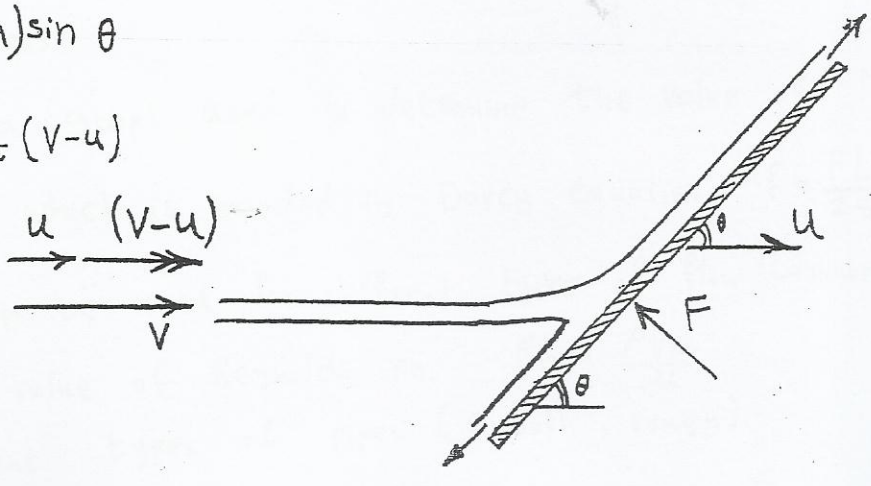


ⓔ

$$\Sigma F = \rho Q(V_f - V_I)$$

$$F = \rho Q\,(V-u)\sin\theta$$

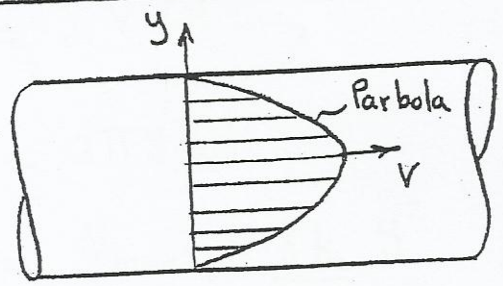
where $Q = A_{jet}(V-u)$



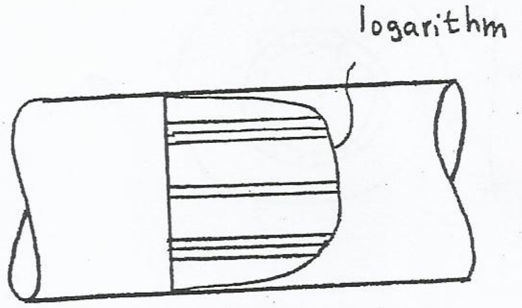
18- Sketch the velocity distribution for a fluid moving in a pipe in case of

i) Laminar flow   ii) Turblent flow

Velocity distribution



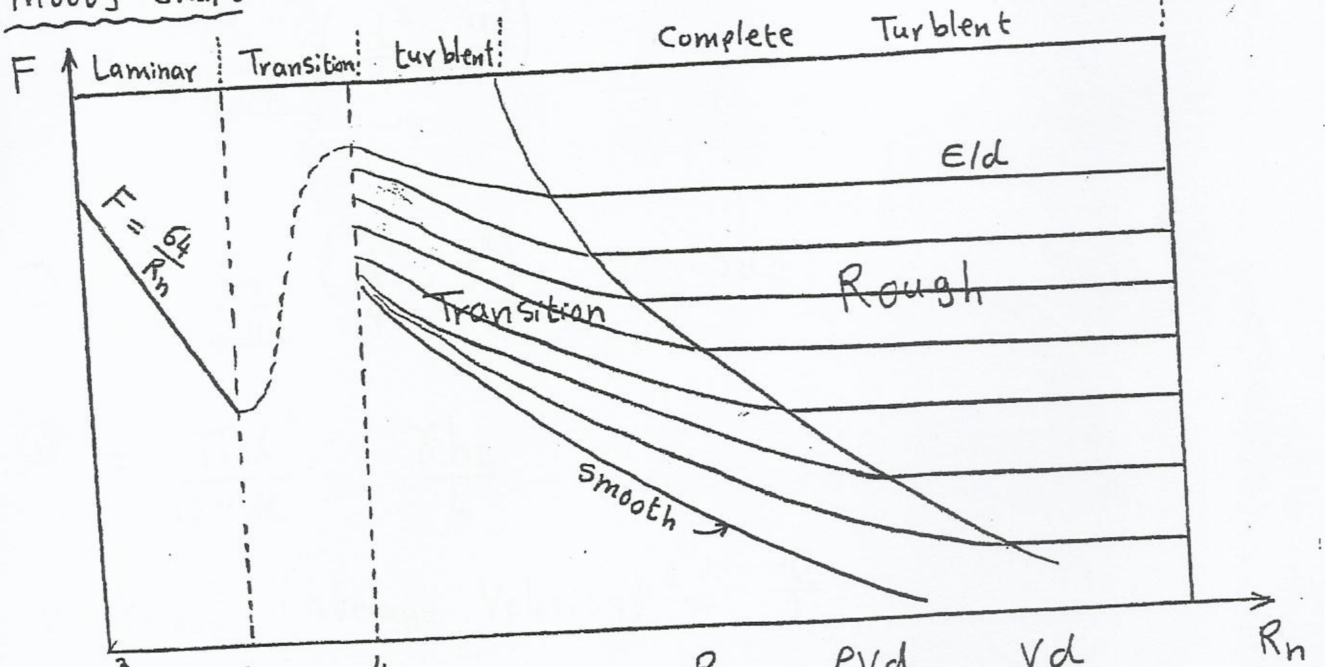
Laminar flow



Turblent flow

19- Sketch Moody Chart

Moody chart



$R_n = \frac{\rho V d}{\mu} = \frac{V d}{\nu}$

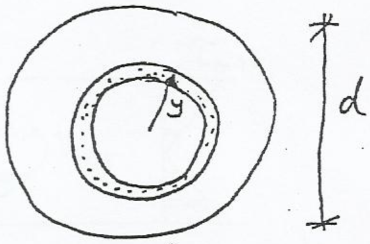
where ϵ = roughness

Moody chart is a graph used to detemine the value of the Friction factor F which is needed in Darcy equation $h_f = \frac{F L V^2}{2 g d}$

* The diagram can be used for diffrent types of flow (Laminar, Turblent) depending on the value of Reynolds no. $R_n = \frac{\rho V d}{\mu}$
* Also for different types of pipes (Smooth, rough) depending on the relative roughness of the pipe $\frac{\epsilon}{d}$

20- Derive an expression for head loss due to friction in pipe for laminar flow

<u>Head loss in pipes</u> (Laminar)



$$Q = \int V \, dA$$

$$dA = 2\pi y \, dy$$

$$V = V_{max} - \frac{\gamma h_L}{4\mu L} y^2$$

$$= \frac{\gamma h_L d^2}{16\mu L} - \frac{\gamma h_L y^2}{4\mu L}$$

$$= \frac{\gamma h_L}{4\mu L}\left(\frac{d^2}{4} - y^2\right)$$

$$Q = \int_0^{D/2} \frac{\gamma h_L}{4\mu L}\left(\frac{d^2}{4} - y^2\right) . \; 2\pi \, y \, dy$$

$$Q = \frac{\pi d^4}{128\mu} . \frac{\gamma h_L}{L}$$

$\because$ V_{mean} (average Velocity) $= \frac{Q}{A}$

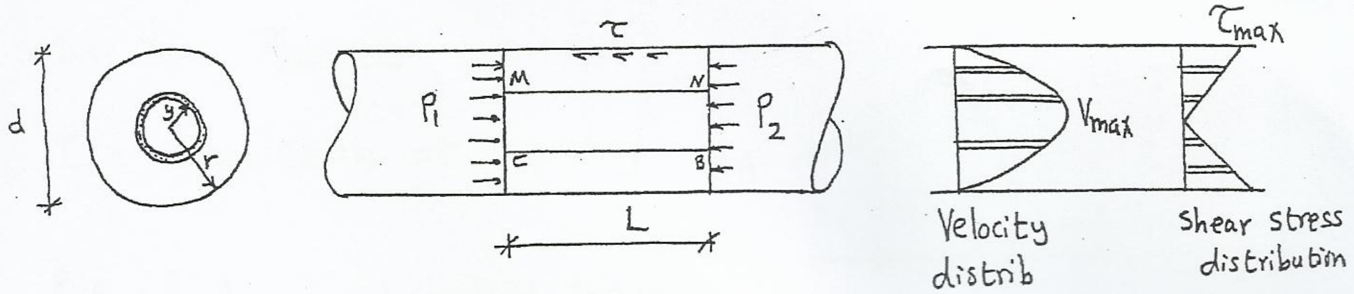
$$V_{mean} = \frac{\cancel{\pi} d^4}{128\mu} . \frac{\gamma h_L}{L} * \frac{4}{\cancel{\pi} d^2} =$$

$$V_{mean} = \frac{d^2 \gamma h_L}{32 \mu L}$$

$$\therefore \; h_L = \frac{32 \mu L V_{mean}}{\gamma d^2}$$

21- Derive an expression for velocity distribution for viscous flow

Velocity and Shear stress distribution in pipes (Laminar)



$$\tau = \mu \frac{dV}{dy}$$

$$F_{shear} = -\mu \frac{dV}{dy} * 2\pi y L \rightarrow ①$$

$$F_{pipe} = (P_1 - P_2)\pi y^2$$

$$= \gamma h_L \pi y^2 \rightarrow ②$$

from ①, ②

$$\mu \frac{dV}{dy} * 2\cancel{\pi}\cancel{y}L = \gamma h_L \cancel{\pi} y^{\cancel{2}}$$

$$dV = -\frac{\gamma h_L}{2\mu L} y\, dy$$

$$\int dV = -\frac{\gamma h_L}{2\mu L} \int y\, dy$$

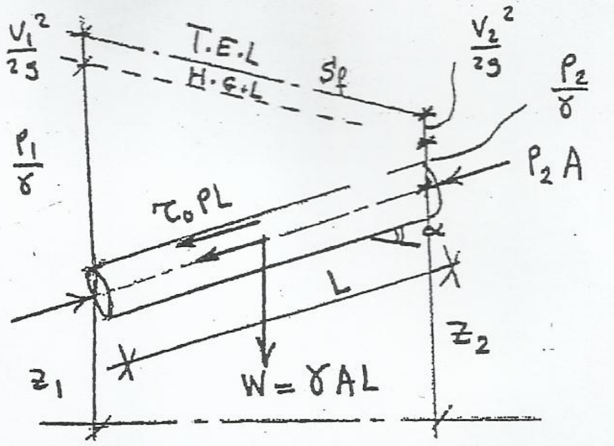
$$V = -\frac{\gamma h_L}{2\mu L} \frac{y^2}{2} + C$$

at $y = 0 \Rightarrow V = V_{max} \Rightarrow C = V_{max}$

$$\boxed{V = V_{max} - \frac{\gamma h_L}{4\mu L} y^2}$$ Velocity at any point

at $y = 0$ (at the center) $$\boxed{V_{max} = \frac{\gamma h_L d^2}{16\mu L}}$$ max velocity

22- Derive an expression for the shear stress in pipe flow



Shear Stress in pipes

$$\sin\alpha = \frac{Z_2 - Z_1}{L}$$

τ_o : Shear Stress at boundary

$$P_1 A - P_2 A - \tau_o P L - \gamma A L \sin\alpha = 0$$

perimeter

$\div \gamma A$

$$\frac{P_1}{\gamma} - \frac{P_2}{\gamma} - \frac{L(Z_2 - Z_1)}{L} = \frac{\tau_o P L}{\gamma A} \quad \text{------}\rightarrow (1)$$

$$h_L = \left(\frac{P_1}{\gamma} + Z_1\right) - \left(\frac{P_2}{\gamma} + Z_2\right) \quad \text{------}\rightarrow (2)$$

$$\therefore h_L = \frac{\tau_o P L}{\gamma A}$$

$$\therefore \boxed{\tau_o = \gamma \frac{A}{P} \frac{h_L}{L} = \gamma R S_f}$$

shear stress at walls of the pipe (or boundaries)

R = Hydraulic radius

$$R = \frac{\text{Area}}{\text{Wetted perimeter}} = \frac{A}{P}$$

in pipes $\quad R = \frac{\pi d^2}{4} \cdot \frac{1}{\pi d} = \frac{d}{4}$

S_f = slope of the T.E.L

$$S_f = \frac{h_L}{L}$$